Reading Seminar: Prismatic F-Gauges

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Schedule

The seminar meets at TBA on TBA in the TBA.

Date		Topic	Speaker
	-1	Filtrations	
Week of 13/04	0	Motivation	
Week of 20/04	1	Algebraic de Rham cohomology vua stacks	
Week of 27/04	2	de Rham stacks and Witt vectors	
Week of 04/05	3	Conjugate filtration	
Week of 01/06	4	Prismatization in characteristic p	
Week of 08/06	5	Filtered prismatization and gagues	
Week of 15/06	6	Syntomification in positive characteristic	
Week of 22/06	7	More on syntomification	
Week of 06/07	8	Compact generation	
Week of 13/07	9	Serre duality	
Week of 20/07	10	Prismatizzation in mixed characteristic	
Week of 27/07	11	Filtered Cartier–Witt divisors	

Schedule of talks

Syllabus

Talk -I: $\mathbf{A}^1/\mathbf{G}_m$ and filtrations

Discuss the equivalence $\mathscr{D}_{\mathrm{qc}}(\mathbf{A}^1/\mathbf{G}_m) \simeq \mathrm{Sp}^{\mathrm{Fil}}$, over the sphere spectrum or a commutative ring as desired. This is a prerequisite to the seminar, so this talk will be a pre-seminar talk for those who need it. The main reference is [Mou21].

Talk 0: Motivation and background + Talk distribution

Discuss Chapter 1 of [BL22]. The first point of this talk is to motivate the category of prismatic F-gauges as a suitable category of coefficients for syntomic cohomology which is of "motivic" nature, in analogy with the category of mixed Hodge modules or the cosntructible derived category of ℓ-adic sheaves. The second point of this talk is to highlight the "geometrisation" programme followed in these notes: following ideas of Simpson, Drinfeld, Bhatt, Lurie etc., these categories of coefficients arise as the quasicoherent derived categories of the transmutation of a scheme by a certain ring stack

Talk I: Algebraic de Rham cohomology via stacks

Discuss [BL22, Section 2.2-2.4]. Note that the discussion of filtrations and $\mathbf{A}^1/\mathbf{G}_m$ can be kept brief as this is now already covered in the pre-seminar talk. Cover the technical discussion on $\mathscr{D}_{\mathrm{qc}}(\mathrm{B}\mathbf{G}_a)$ in characteristic zero and use this to construct the filtered de Rham stack in characteristic zero. Then discuss what happens in mixed characteristic and how the divided power hull of $\mathbf{G}_a$ shows up.

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Talk II: de Rham stacks and Witt vectors

Discuss [BL22, Section 2.5-2.6]. First, construct the dr Rham stack in mixed characteristic using the results from the previous talk, and conclude by proving the crystalline miracle using the stacky perspective. Then discuss the relation to realisation of $\mathbf{G}_a^\sharp$ via Witt vectors, i.e. using the relation between divided powers and ∂-structures.

Talk III: Conjugate filtration

Discuss [BL22, Section 2.7-2.8]. Construct the conjugate filtration in characteristic p using our stacky perspective and identifying the de Rham stack as a gerbe. If time permits, discuss some criteria for the splitting of this gerbe and the relation to the Deligne–Illusie theorem. Conclude by gluing together the Hodge and conjugate filtrations (not every result in this section has a full proof).

Talk IV: Prismatization in characteristic p

Discuss [BL22, Section 3.1-3.2]. This consists of some recollections on absolute prismatization over a perfect field of characteristic p and the Nygaard filtration. If necessary, supplement with background material from [Bha22].

Talk V: Filtered prismatization and gauges

Discuss [BL22, Section 3.3-3.4], and note that we will skip [BL22, Section 3.5]. Construt the Nygaard filtration by transmutation and use this to describe gauges over k.

Talk VI: Syntomification in positive characteristic

Discuss [BL22, Section 4.1-4.2]. Construct the syntomification in positive characteristic, and present a general discussion of its quasicoherent derived category

Talk VII: More on syntomification

Building on the previous talk, discuss [BL22, Section 4.3-4.4]. Discuss vector bundles on the syntomification of k and proceed to define syntomic cohomology and discuss its basic properties.

Talk VIII: Compact generation of prismatic F-Gauges

Time for an intermezzo: Discuss [CF25, Section 9.1] which resolves the question in [BL22, Remark 4.4.6]

Talk IX: Serre duality

Discuss [BL22, Section 4.5], i.e. the proof of Serre duality on the syntomification of $\mathbf{F}_p$.

Talk X: Prismatization in mixed characteristic

Generalise the constructions from the previous talks to mixed characteristic: discuss [BL22, Section 5.1-5.2]. Discuss admissible W-modules and some of the computations that go behind this cosntruction in preparation for the next talk.

Talk XI: Filtered Cartier–Witt divisors

Discuss [BL22, Section 5.3-5.5] which introduces filtered Cartier–Witt divisoers over p-nilpotent rings. This involves a compatibility check with the previous definitions in characteristic p and a more explicit description of what happens in the quasiregular semiperfectoid case.

References

[Bha22] Bhargav Bhatt. "Absolute prismatic cohomology". *arXiv preprint arXiv:2201.06120* (2022).

[BL22] Bhargav Bhatt and Jacob Lurie. *Prismatic F-gauges*. 2022. URL: https://www.math.ias.edu/~bhatt/teaching/mat549f22/lectures.pdf.

[CF25] Shachar Carmeli and Tony Feng. "Prismatic Steenrod operations and arithmetic duality on Brauer groups". *arXiv preprint arXiv:2507.13471* (2025).

[Mou21] Tasos Moulinos. "The geometry of filtrations". *Bulletin of the London Mathematical Society* 53.5 (2021), pp. 1486–1499.